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Chew Lian Chua and Sandy Suardi

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Chew Lian Chua† and Sandy Suardi§
†Melbourne Institute of Applied Economic and Social Research, The University of Melbourne
§School of Economics, The University of Queensland

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Melbourne Institute of Applied Economic and Social Research
The University of Melbourne
Victoria 3010 Australia
***Telephone* (03) 8344 2100**
***Fax* (03) 8344 2111**
***Email* melb-inst@unimelb.edu.au**
***WWW Address* http://www.melbourneinstitute.com**

Abstract

The use of GARCH and jump models to capture asset price dynamics is ubiquitous in economics and finance literature. We show that the size of Breitung (2002) nonparametric unit root test is robust to the presence of jump and GARCH errors but not for the other standard unit root tests. The power performance of all tests, except for Phillips (1987) test, is fairly robust provided that the mean process is not nearly integrated.

Keywords: Unit Root; Jumps; GARCH

J.E.L. Reference Numbers: C3

1. Introduction

This paper examines the size and power properties of several unit root tests when applied to data that are generated from a jump diffusion process with GARCH errors. This issue is especially relevant given that many high frequency financial time series are characterized by non-stationarity and their errors are characterized by jump processes and conditional heteroskedasticity. There is a growing body of literature that uses jump models to characterize the dynamics of numerous financial data such as stock returns (Ball and Torous, 1983; Chan and Maheu, 2002), exchange rates (Jorion, 1993; Vlaar and Palm, 1993), interest rates (Naik and Lee, 1993; Das, 2002), and electricity prices (Johnson and Barz, 1999; Knittel and Roberts, 2001). These jump models are often combined with GARCH errors because they more adequately capture the leptokurtosis commonly observed in the unconditional distribution of financial data. Jump models are also better suited to explain large discrete changes found in asset returns widely observed in speculative markets than GARCH and stochastic volatility models (see Gallant, et al. 1997; Andersen et al. 1999).

The literature on unit root test has exclusively focused on the effects of GARCH error processes on the performance of standard unit root tests. The GARCH process of Bollerslev (1986), that permit a class of time series models for which the conditional variance is allowed to vary through time as a function of current and past information, clearly violates the constant variance assumption of many unit root tests. Kim and Schmidt (1993) and Haldrup (1994), amongst others, have shown that standard Dickey Fuller (DF) tests are subject to minor size and power distortions provided that the variance process is not degenerate (i.e. the intercept of the conditional variance specification is not zero) and the volatility parameter (i.e. the coefficient of the squared and lagged residuals) is not far from zero. This note extends the unit root test literature by taking into account the effects of jump processes and GARCH errors on the performance of unit root tests.

In the absence of asymptotic theory for unit root tests in the presence of jump and GARCH errors, Monte Carlo experiments are employed to investigate the reliability of the standard DF test, Phillips (1987) semiparametric test, the heteroskedasticity-robust DF test using White's corrected standard errors, and Breitung (2002) nonparametric test when applied to data that exhibit jump and GARCH errors. The former two tests are commonly employed in econometric modelling to determine the mean-reversion property of the data. The rest of the paper is organised as follows. Section 2 discusses the four types of unit root tests. Section 3 lays out the design and conduct of the experiment. It also discusses the results and their implications for empirical research. Section 4 concludes.

2. Testing the Unit Root Null in the Mean

Consider a data generating process (DGP) that follows a jump process with GARCH errors

$$y_t = \alpha + \beta y_{t-1} + u_t + J_t \cdot I_t \quad (1)$$

where $u_t \mid \Omega_{t-1} \sim N(0, h_t)$, $u_t = z_t \sqrt{h_t}$, $z_t \sim N(0,1)$ and $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$ is a GARCH(1,1) process. The jump component $J_t \sim N(0, \delta^2)$ and I_t is drawn from a Poisson distribution such that $P(I_t = j \mid \Omega_{t-1}) = \exp(-\lambda)\lambda^j / j\,!$ for $j = 0,1,2,...$ and λ is the jump intensity. This model is a simplification of many financial models that may include more complex structures in the conditional mean and/ or conditional variance equations. The null hypothesis of a unit root (i.e. $H_0 : \beta = 1$) suggests that y_t degenerates at zero asymptotically.

We consider four different types of unit root tests for which two of them are regarded as popular testing procedures that are commonly reported in empirical research. They are the Dickey and Fuller (1979) (DF) and Phillips (1987) (PP) semi-parametrically corrected tests. The standard DF test statistics are obtained by running the OLS regression

$$\Delta y_t = \alpha + \beta y_{t-1} + e_t \quad (2)$$

where $e_t \sim i.i.d\,(0,1)$ by assumption. Note that the presence of the GARCH error term would violate the i.i.d assumption of the DF regression. The DF test statistic is either a one-sided t-ratio for the significance of β or a joint F-test for the joint hypothesis $\alpha = \beta = 0$. We only consider the case of $\alpha = 0$ for ease of exposition. Since heteroskedasticity in stationary time series models will yield invalid inference, one strategy is to employ White's (1980) corrected standard errors when computing the DF t-ratio (see Haldrup, 1994; Kim and Schmidt, 1993). This heteroskedasticity corrected t-ratio, which we refer to as DFW, merits investigation because it is a commonly employed correction in non-stationary time series models.[1] Kim

[1] Apart from unit root considerations, the asymptotic theory for the White correction requires the existence of both the variance and the fourth moment of the error. The condition for the existence of the unconditional fourth moment for GARCH(1,1) is $3\phi_1^2 + 2\phi_1\phi_2 + \phi_2^2 < 1$ (see Bollerslev, 1986). This condition is satisfied for the parameter values considered in our simulation.

and Schmidt (1993) show the White standard correction improves the accuracy of the DF tests faily well in the integrated and degenerate case.

Phillips (1987) suggests correcting the t-ratio test statistic non-parametrically when e_t is weakly dependent. The modified PP test statistic is

$$Z(t_\beta) = (s_u / s_T)t_\beta - \left(\tfrac{1}{2}\right)(s_T^2 - s_u^2)\left\{ s_T \left[T^{-2}\sum_{t=1}^{T}(y_{t-1} - \overline{y}_{t-1})^2 \right]^{1/2} \right\}^{-1} \tag{3}$$

with s_u^2 and s_T^2 being innovation variance estimators given by $s_u^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_t^2$ and $s_T^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_t^2 + 2T^{-1}\sum_{j=1}^{T-1}k(j/\xi_T)\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}$ where k(.) is a kernel and ξ_T is its bandwidth parameter. The Bartlett kernel which possesses a highly desirable property of ensuring nonnegativity of the variance estimate (Newey and West, 1987) is widely used to compute the PP test statistic.

Lastly, we consider Breitung (2002) nonparametric (NP) test that is shown to be robust to GARCH errors even under integrated or explosive volatility. Such a property is generally not satisfied by the standard DF and PP tests. Breitung's NP test is defined as

$$NP = \frac{T^{-2}\sum_{t=1}^{T}\hat{U}_t^2}{\sum_{t=1}^{T}\hat{u}_t^2} \tag{4}$$

where $\hat{U} = \sum_{i=1}^{t}\hat{u}_i$, $\hat{u}_i = y_i - \overline{y}$ where $\overline{y}$ is the sample mean. The null of a unit root is rejected when the value of the variance-ratio statistic is lower than the respective critical values reported in Table 5 of Breitung (2002).

3. Monte Carlo Experiment

3.1 Simulation Design

The finite-sample properties (size and power) of the tests described in the previous section are examined in the presence of jump process and its effects with GARCH errors. We first examine the empirical size of the tests by simulating data from (1) with β=1.0. In setting the parameter values, we consider how close the GARCH process is to being integrated, the size of the volatility parameters (ϕ_1), and the size (δ) and intensity (λ) of the jump. To this end, we consider $\phi_1+\phi_2=$ (0.9,0.99) by setting ϕ_1=0.1 with ϕ_2=(0.8,0.89), and ϕ_1=0.5 with ϕ_2=(0.4,0.49).[2] We set ϕ_0=1 so that as $\phi_1+\phi_2$ tends to one the unconditional variance increases and there is no degeneracy. The size and frequency of jumps are controlled by δ = (0.01, 0.1) and λ = (0.01, 0.5).[3] To mitigate the effects of start-up values, we discard the initial 500 observations. For empirical relevance, samples of 250 and 500 observations are drawn and the experiment is repeated 10,000 times. The empirical power of the tests is examined for β between 0.95 and 0.99 with 0.01 increments. The near-integrated region in the mean warrants investigation as empirical evidence suggests that many financial time series, such as short-term interest rates (Das, 2002), exhibit near unit root property.

3.2 Results

Empirical size and power with jumps

Table 1 reports the results on the size of the tests when the DGP contains a jump process. There are moderate deviations from the nominal size for all tests in all cases. It is not surprising to find that the heteroskedasticity-consistent covariance matrix proposed by White (1980) does not correct for the bias generated by the jump process. Instead the empirical size of DFW test is marginally inflated compared with that of the DF test. For the size-adjusted power of the tests we only report results for T=500, $\delta = 0.1$ and $\lambda = 0.01$ as the results for T=250 and other values of δ and λ are by and large consistent with the reported case. Figure 1 shows that the power of the tests declines sharply as the mean process becomes more integrated (i.e. as β approaches unity) at both 1% and 5% significance levels. We find that the NP test's power performs worse than its parametric (DF and DFW) and semi-parametric (PP) counterparts.

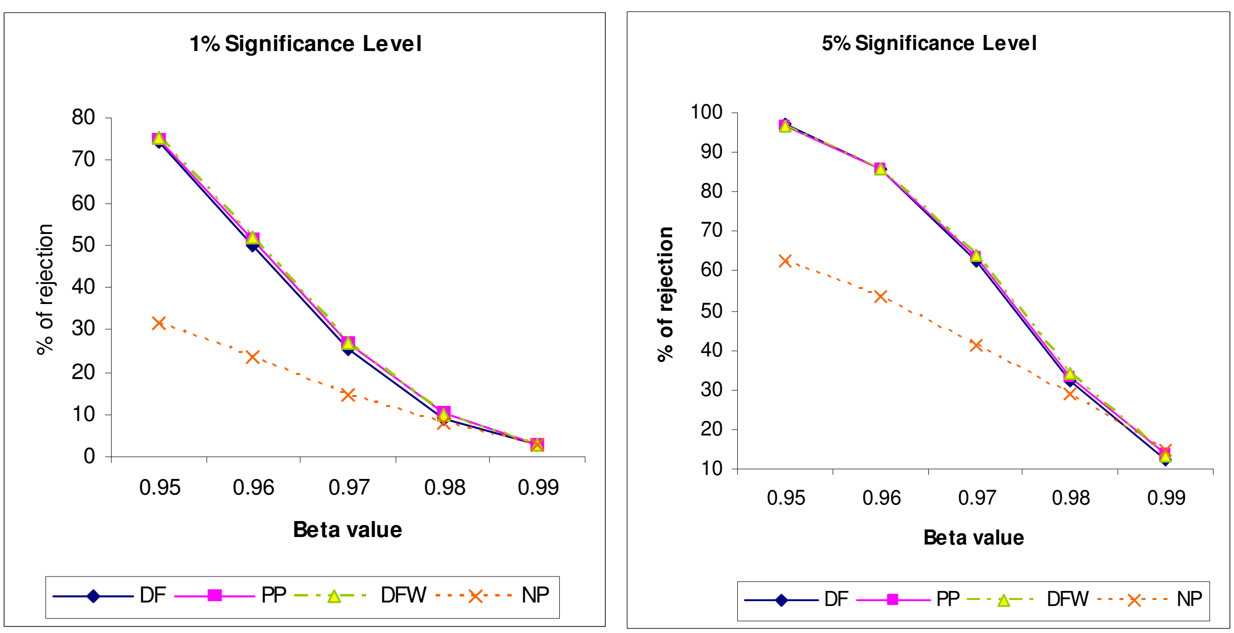
[2] Many financial time series are defined by a near-integrated GARCH process. See Kim and Schmidt (1993) for a review of the parametric configurations that arise in empirical question.
[3] The jump size and frequency values are chosen to be consistent with those reported in the literature.

Table 1. Empirical sizes in the presence of a jump process (T=500)

(δ, λ)	Tests	1%	5%	10%
(0.01,0.01)	DF	1.19	5.38	9.98
	PP	1.37	5.65	10.42
	DFW	1.32	5.65	10.48
	NP	0.86	5.72	10.79
(0.01,0.5)	DF	1.16	5.21	10.52
	PP	1.38	5.83	10.91
	DFW	1.26	5.58	10.58
	NP	0.9	5.61	10.92
(0.1,0.01)	DF	0.97	5.2	10.22
	PP	1.16	5.66	10.64
	DFW	1.17	5.41	10.73
	NP	0.94	5.72	10.98
(0.1,0.5)	DF	1.27	5.03	9.92
	PP	1.38	5.5	10.19
	DFW	1.22	5.39	10.03
	NP	0.77	4.98	10.54

Note: The tests are Dickey-Fuller t-test (DF), Phillip's (1987) modified t-test (PP), White's corrected Dickey-Fuller test (DFW) and Breitung's (2002) test (NP).

Figure 1. Power under different values of β with $\delta = 0.1$, $\lambda = 0.01$, and T=500



Note: See note to Table 1.

Empirical size and power with jumps and GARCH errors

From table 2 we can observe that only the empirical size of NP test displays robustness against jump process and GARCH errors. Both the empirical size of DF and DFW tests are biased upward, while that of PP test is biased downward. Taken together, the results suggest that the DF and DFW (PP) tests would tend to over (under) reject the null of a unit root, while the NP test would provide a more accurate unit root diagnostic for a series that exhibits both jumps and GARCH errors. Again, we find that the White's heteroskedastic correction does not seem to work well at decreasing the empirical size of the DF test when a jump process is present with GARCH errors. There is evidence that as the jump size or jump intensity increases (holding all other parameters constant), the empirical size increases marginally. In addition, as the GARCH process becomes more integrated (i.e. $\phi_1 + \phi_2$ approaches 1) the empirical size of DF and DFW (PP) tests increases (decreases). An

Table 2. Empirical sizes in the presence of a jump process and GARCH errors (T=500)

	1%				5%			
DF	$\phi_1 + \phi_2 = 0.90$		$\phi_1 + \phi_2 = 0.99$		$\phi_1 + \phi_2 = 0.90$		$\phi_1 + \phi_2 = 0.99$	
(δ, λ)	(0.1,0.8)	(0.5,0.4)	(0.1,0.89)	(0.5,0.49)	(0.1,0.8)	(0.5,0.4)	(0.1,0.89)	(0.5,0.49)
(0.01,0.01)	2.98	2.26	4.42	4.47	8.45	6.97	10.15	10.09
(0.01,0.5)	3.41	2.28	4.53	4.49	8.47	7.24	10.26	10.13
(0.1,0.01)	3.25	2.29	4.46	4.61	8.48	6.96	10.31	10.24
(0.1,0.5)	3.42	2.43	4.56	4.63	8.51	7.50	10.38	10.27
PP								
(0.01,0.01)	0.51	0.82	0.42	0.61	3.37	4.39	2.84	3.21
(0.01,0.5)	0.57	0.84	0.51	0.72	3.39	4.48	3.13	3.30
(0.1,0.01)	0.65	0.84	0.53	0.67	3.48	3.95	3.28	3.71
(0.1,0.5)	0.67	0.89	0.63	0.75	3.50	4.50	3.35	3.74
DFW								
(0.01,0.01)	3.83	2.35	5.28	5.37	9.44	7.48	11.06	11.71
(0.01,0.5)	4.14	2.46	5.82	5.46	9.52	7.86	11.90	11.77
(0.1,0.01)	4.19	2.38	5.62	5.82	9.67	7.50	11.78	11.86
(0.1,0.5)	4.21	2.83	5.85	5.88	9.80	8.09	11.98	11.89
NP								
(0.01,0.01)	0.72	0.90	0.91	1.01	4.77	5.15	5.23	5.40
(0.01,0.5)	0.90	0.91	0.95	1.07	4.95	5.47	5.38	5.55
(0.1,0.01)	0.91	0.92	0.92	1.13	5.09	5.40	5.44	5.45
(0.1,0.5)	0.92	0.95	0.97	1.15	5.45	5.87	5.89	5.92

Note: See note to Table 1.

Table 3. Power at 5% nominal level in the presence of a jump process and GARCH errors (T=500)

	$\beta = 0.95$				$\beta = 0.99$			
DF	$\phi_1 + \phi_2 = 0.90$		$\phi_1 + \phi_2 = 0.99$		$\phi_1 + \phi_2 = 0.90$		$\phi_1 + \phi_2 = 0.99$	
(δ, λ)	(0.1,0.8)	(0.5,0.4)	(0.1,0.89)	(0.5,0.49)	(0.1,0.8)	(0.5,0.4)	(0.1,0.89)	(0.5,0.49)
(0.01,0.01)	90.92	93.05	90.73	90.84	18.63	19.18	20.92	21.51
(0.01,0.5)	91.52	93.35	89.87	90.99	18.62	19.26	21.08	21.84
(0.1,0.01)	91.52	93.40	90.17	90.66	19.32	19.72	20.82	21.80
(0.1,0.5)	91.20	93.01	90.03	90.59	19.10	19.83	20.81	20.97
PP								
(0.01,0.01)	47.65	50.36	30.46	39.12	8.96	9.35	8.28	8.38
(0.01,0.5)	48.94	50.43	39.33	39.66	8.86	9.24	7.92	8.32
(0.1,0.01)	48.95	50.39	38.87	39.95	8.95	9.91	8.17	8.24
(0.1,0.5)	48.66	50.84	38.91	40.62	9.39	9.92	8.21	8.47
DFW								
(0.01,0.01)	90.02	92.25	89.19	89.26	20.12	20.95	23.43	23.86
(0.01,0.5)	90.62	92.42	88.22	89.42	20.40	20.51	23.49	23.57
(0.1,0.01)	90.71	92.65	88.24	88.99	20.55	20.86	23.22	23.88
(0.1,0.5)	90.49	92.19	88.61	88.66	20.66	20.98	23.21	23.99
NP								
(0.01,0.01)	62.22	62.75	61.93	62.42	13.78	14.21	13.59	13.67
(0.01,0.5)	62.24	62.26	62.29	62.45	14.01	14.29	13.78	13.91
(0.1,0.01)	62.33	62.51	62.30	62.57	13.94	14.58	13.49	13.61
(0.1,0.5)	62.50	62.33	61.42	62.65	13.79	14.16	13.14	13.98

Note: See note to Table 1

increase in the volatility paramater ϕ_1 leads to a marginal decrease (increase) in the empirical size of both the DF and DFW (PP and NP) tests.

Referring to table 3, it can be seen that the PP test has the lowest size-adjusted power relative to the other tests. While the DF and DFW tests' power is robust against jump and GARCH errors for $\beta = 0.95$, their power quickly deteriorates when the root of the mean process is close to unity.[4] The power of NP and PP tests also reduces sharply in the near-integrated region of the mean process. We further investigate the power performance for a large sample size of T=1000 and find that there is improvement in the power function but in the case of a

[4] We do not report the results for $\beta = 0.96$ to 0.98 due to space constraint. We find that the power of all the tests deteriorates as β increases. These results are available from the authors upon request.

near-integrated mean process the deterioration in all the tests' power remains severe.[5] As the GARCH process becomes more integrated, we observe significant (moderate) reduction in the power of the PP (NP) tests. However, for both the DF and DFW tests, there is mild improvement in their power when the mean and GARCH processes become more integrated. There is no discernible pattern regarding the impact of jump intensity or size on the power of the tests.

4. Conclusion

We examine the performance of various unit root tests in the presence of a jump process with GARCH errors. The empirical sizes of these tests are fairly robust against jump process but their power generally suffers from severe distortion as the mean process becomes near-integrated. In the presence of both jump and GARCH errors, all tests, apart from Breitung (2002) NP test, suffer from size distortion. The power performance of Dickey-Fuller and White (1980) heteroskedasticity corrected Dickey-Fuller tests are robust to both jump and GARCH errors provided that the mean process is not close to being integrated. Overall, our Monte Carlo results suggest that we should be suspicious of the accuracy of most of these tests when a jump process and GARCH errors are present in the data, and when the mean process is nearly integrated.

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[5] For brevity, we do not present these results, but these are available from the authors upon request.

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